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Electrodynamics on matrix space: non-Abelian by coordinates

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Abstract. We consider the dynamics of a charged particle in a space whose coordinates are $N\times N$ hermitian matrices. Putting things in the framework of D0-branes of string theory, we mention that the transformations of the matrix coordinates induce non-Abelian transformations on the gauge potentials. The Lorentz equations of motion for matrix coordinates are derived, and it is observed that the field strengths transform like their non-Abelian counterparts. The issue of the map between theory on matrix space and ordinary non-Abelian gauge theory is discussed. The phenomenological aspect of "finite-N non-commutativity" for the bound states of D0-branes appears to be very attractive.

1 Electrodynamics on matrix space

We begin with the dynamics of a charged point particle in a space whose coordinates are $N \times N$ hermitian matrices, such as

$$X^i = X^i_a T^a, \quad i = 1, \cdots, d, \quad a = 1, \cdots, N^2 \tag{1}$$

in which the T^a are the basis for the hermitian matrices (i.e., the generators of $U(N)$). The action may be in the form of

$$S[X] = \int \mathrm{d}t\mathrm{Tr}(\frac{1}{2}m\dot{X}_i\dot{X}^i + q\dot{X}^i A_i(X,t) \\ -qA_t(X,t) - V(X)), \tag{2}$$

which can be obtained simply by replacing the ordinary coordinates, x, by their matrix form X, in the action $S[x] = \int \mathrm{d}t(\frac{1}{2}m\dot{x}_i\dot{x}^i - q\dot{x}^i A_i(x,t) - qA_t(x,t) - V(x))$, simply adding a "Tr" on the matrix structure. Besides we assume that the gauge potentials $(A_t(X,t), A_i(X,t))$ have a functional dependence on the matrix coordinates X, and to put things simple (and natural) the Tr should be calculated by a "symmetrization prescription" on the matrices X. By a symmetrization prescription we mean symmetrization on all of the X's appearing in the potentials; this can be obtained by the so-called "non-Abelian Taylor expansion,"

$$A_\mu(X,t) = A_\mu(x,t)|_{x\to X} \equiv \exp[X^i\partial_{x^i}]A_\mu(x,t) \tag{3}$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} X^{i_1} \cdots X^{i_n} (\partial_{x^{i_1}} \cdots \partial_{x^{i_n}}) A_\mu(x,t)|_{x=0},$$

with $\mu = t, i$. In the above expansion the symmetrization is recovered via the symmetric property of the derivatives inside the term $(\partial_{x^{i_1}} \cdots \partial_{x^{i_n}})$. Now we have an action with enhanced degrees of freedom, from d in ordinary space, to $d \times N^2$ in the space with matrix coordinates.

The fate of the $U(1)$ symmetry of the action $S[x]$, with transformations

$$A_\mu(x,t) \to A'_\mu(x,t) = A_\mu(x,t) - \partial_\mu \Lambda(x,t), \tag{4}$$

in the new action $S[X]$ is interesting. One can see that the action $S[X]$ is also symmetric under similar transformations,

$$A_t(X,t) \to A'_t(X,t) = A_t(X,t) - \partial_t \Lambda(X,t) \\ A_i(X,t) \to A'_i(X,t) = A_i(X,t) + \delta_i \Lambda(X,t), \tag{5}$$

in which δ_i is the functional derivative $\delta/\delta X^i$. Consequently one obtains

$$\delta S[X] \sim q \int \mathrm{d}t\mathrm{Tr}(\partial_t \Lambda(X,t) + \dot{X}^i \delta_i \Lambda(X,t)) \\ \sim q \int \mathrm{d}t\mathrm{Tr}\left(\frac{\mathrm{d}\Lambda(X,t)}{\mathrm{d}t}\right) \sim 0. \tag{6}$$

2 D0-brane picture

Since we are performing the symmetrization in the gauge potentials (A_t, A_i), the symmetric parts of the potential $V(X)$ can be absorbed in a redefinition of $A_t(X,t)$. So

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the interesting parts of $V(X)$ contain "commutators" of coordinates, in an expansion they could be presented by

$$V(X) = \underbrace{[X^i, X^j] + X^i[X^j, X^k]}_{\text{traceless or unsummed index}} - m\frac{[X^i, X^j]^2}{l^4} + O(X^6)\cdots, \tag{7}$$

in which l is a parameter with dimension of length. Consequently, the action (2) will be found to be the (low-energy bosonic) action of N D0-branes in a 1-form RR field background (A_t, A_i), in the "temporal gauge" $a_0(t) = 0$. From the string theory point of view, D0-branes are point particles to which ends of strings are attached [1,2]. In a bound state of N D0-branes, D0-branes are connected to each other by strings stretched between them, and it can be shown that the correct dynamical variables describing the positions of D0-branes, rather than numbers, are $N \times N$ hermitian matrices [3]. By restoring the (world-line) gauge potential $a_0(t)$, we end up with the action [4,5]

$$S_{\mathrm{D0}} = \int \mathrm{d}t\mathrm{Tr}\left(\frac{1}{2}mD_tX_iD_tX^i - qD_tX^iA_i(X,t) - qA_t(X,t) + m\frac{[X^i, X^j]^2}{l^4} + \cdots\right), \tag{8}$$

with $D_t = \partial_t + \mathrm{i}[a_0(t),\ ]$ as covariant derivative. Ignoring for the moment the gauge potentials (A_t, A_i), the equations of motion can be solved by diagonal configurations, such as

$$X^i(t) = \mathrm{diag.}(x^i_1(t), \cdots, x^i_N(t)),$$
$$a_0(t) = \mathrm{diag.}(a_{01}(t), \cdots, a_{0N}(t)), \tag{9}$$

with $x^i_\alpha(t) = x^i_{\alpha 0} + v^i_\alpha t$, $\alpha = 1, \cdots, N$. By this configuration, we restrict the $U(N)$ generators T^a to the N dimensional Cartan (diagonal) sub-algebra; the symmetry is broken from $U(N)$ to $U(1)^N$. This configuration describes the classical free motion of N D0-branes, neglecting the effects of the strings stretched between them. Of course the situation is different when we consider quantum effects, and consequently it will be found that the dynamics of the off-diagonal elements capture the oscillations of the stretched strings.

It can be seen that the transformations (5) also leave the action (8) invariant. By replacements one finds [6]

$$\delta S_{\mathrm{D0}} \sim \delta S[X] + q\int \mathrm{d}t\mathrm{Tr}(\mathrm{i}a_0[X^i, \delta_i\Lambda(X,t)]) = 0. \tag{10}$$

$\delta S[X]$ is the expression introduced in (6), and the second term vanishes by the symmetrization prescription [6].

3 Non-Abelian transformations

Actually, the action (8) is invariant under the transformations

$$X^i \to \tilde{X}^i = U^\dagger(X,t)X^iU(X,t),$$
$$a_0(t) \to \tilde{a}_0(X,t) = U^\dagger(X,t)a_0(t)U(X,t) - \mathrm{i}U^\dagger(X,t)\partial_tU(X,t), \tag{11}$$

with $U(X,t)$ an arbitrary $N \times N$ unitary matrix; in fact under these transformations one obtains

$$D_tX^i \to \tilde{D}_t\tilde{X}^i = U^\dagger(X,t)D_tX^iU(X,t), \tag{12}$$
$$D_tD_tX^i \to \tilde{D}_t\tilde{D}_t\tilde{X}^i = U^\dagger(X,t)D_tD_tX^iU(X,t). \tag{13}$$

Now, in the same spirit as for the previously introduced $U(1)$ symmetry of (5), one finds the symmetry transformations

$$X^i \to \tilde{X}^i = U^\dagger(X,t)X^iU(X,t),$$
$$a_0(t) \to \tilde{a}_0(X,t) = U^\dagger(X,t)a_0(t)U(X,t) - \mathrm{i}U^\dagger(X,t)\partial_tU(X,t),$$
$$A_i(X,t) \to \tilde{A}_i(X,t) = U^\dagger(X,t)A_i(X,t)U(X,t) + \mathrm{i}U^\dagger(X,t)\delta_iU(X,t),$$
$$A_t(X,t) \to \tilde{A}_t(X,t) = U^\dagger(X,t)A_t(X,t)U(X,t) - \mathrm{i}U^\dagger(X,t)\partial_tU(X,t), \tag{14}$$

in which we assume that $U(X,t) = \exp(-\mathrm{i}\Lambda)$ is arbitrary up to the condition that $\Lambda(X,t)$ is totally symmetrized in the X's. The above transformations of the gauge potentials are similar to those of non-Abelian gauge theories, and we mention that it is just a consequence of the enhancement of the degrees of freedom going from numbers (x) to matrices (X). In other words, we are faced with a situation in which "the rotation of fields" is generated by "the rotation of coordinates."

The above observation on gauge theory associated to D0-brane matrix coordinates by itself is not a new one, and we already know another example of this kind in noncommutative gauge theories. In spaces whose coordinates satisfy the algebra

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}, \tag{15}$$

with constant $\theta^{\mu\nu}$, the symmetry transformations of the $U(1)$ gauge theory are like those of non-Abelian gauge theory [7–9]; in explicit form

$$A_\mu(x) \to A'_\mu(x) = U^\dagger(x) \star A_\mu(x) \star U(x) - \mathrm{i}U^\dagger(x) \star \partial_\mu U(x), \tag{16}$$

in which the $\star$-products are recognized. Also, one could put things in the reverse direction that we had above for D0-branes. The coordinates x^μ can be transformed locally by the large symmetry of the space by[1] $\tilde{x}^\mu \equiv U(x) \star x^\mu \star U^\dagger(x)$. Note that the above commutation relation is satisfied also by the transformed coordinates. Now, by combining the gauge transformations with a transformation of coordinates one can bring the transformations of gauge fields to the form of a $U(1)$ theory, by

$$x^\mu \to \tilde{x}^\mu = U(x) \star x^\mu \star U^\dagger(x),$$
$$A_\mu(x) \to \tilde{A}_\mu(\tilde{x}) = A'(\tilde{x}) = A_\mu(x) - \partial_\mu\Lambda(x), \tag{17}$$

[1] Here, we are using $\hat{x}$ for operators as coordinates, and x as numbers multiplied by the $\star$-products

with $U = \exp(-\mathrm{i}\Lambda)$. One also notes that by the above transformation the so-called "covariant coordinates" $\mathcal{X}^\mu \equiv x^\mu + (\theta^{-1}A(x))^\mu$ remain invariant. In addition, the case we see here for D0-branes may be considered as another example of the relation between gauge symmetry transformations and transformations of matrix coordinates [10].

The last notable points concern the behavior of $a_0(t)$ and $A_t(X,t)$ under the symmetry transformations (14). From the world-line theory point of view, $a_0(t)$ is a dynamical variable, but $A_t(X,t)$ should be treated as part of the background; however, they behave similarly under transformations. Also we see by (14) that the time, and only the time dependence of $a_0(t)$, which is the consequence of dimensional reduction, should be understood up to a gauge transformation. In [6] a possible map between the dynamics of D0-branes, and the semi-classical dynamics of charged particles in a Yang–Mills background was mentioned. It is worth mentioning that via this possible relation, an explanation for the above notable points can be recognized [6].

4 Lorentz equations of motion

The equations of motion by the action (8), ignoring for the moment the potential term $V(X)$, will be found to be

$$m D_t D_t X_i = q(E_i(X,t) + \underbrace{D_t X^j B_{ji}(X,t)}), \quad (18)$$

$$m[X_i, D_t X^i] = +q[A_i(X,t), X^i], \quad (19)$$

with the following definitions:

$$E_i(X,t) \equiv -\delta_i A_t(X,t) - \partial_t A_i(X,t), \quad (20)$$

$$B_{ji}(X,t) \equiv -\delta_j A_i(X,t) + \delta_i A_j(X,t). \quad (21)$$

Here the symbol $\underbrace{D_t X^j B_{ji}(X,t)}$ denotes the average over all of the positions of $D_t X^j$ between the X's of $B_{ji}(X,t)$. The above equations for the X's are like the Lorentz equations of motion, with the exception that the two sides are $N \times N$ matrices, and the time derivatives ∂_t are replaced by their covariant counterpart D_t [2].

The behavior of (18) and (19) under gauge transformation (14) can be checked. Since the action is invariant under (14), it is expected that the equations of motion change covariantly. The left-hand side of (18) changes to $U^\dagger D_t D_t X U$ by (13), and therefore we should find the same change for the right-hand side. This is in fact the case, since

$$\begin{aligned} f(X,t) &\to \tilde{f}(\tilde{X},t) = U^\dagger(X,t) f(X,t) U(X,t), \\ \delta_i f(X,t) &\to \tilde{\delta}_i \tilde{f}(\tilde{X},t) = U^\dagger(X,t) \delta_i f(X,t) U(X,t), \\ \partial_t f(X,t) &\to \partial_t \tilde{f}(\tilde{X},t) = U^\dagger(X,t) \partial_t f(X,t) U(X,t). \end{aligned} \quad (22)$$

In conclusion, the definitions (20) and (21) lead to

$$\begin{aligned} E_i(X,t) &\to \tilde{E}_i(\tilde{X},t) = U^\dagger(X,t) E_i(X,t) U(X,t), \\ B_{ji}(X,t) &\to \tilde{B}_{ji}(\tilde{X},t) = U^\dagger(X,t) B_{ji}(X,t) U(X,t), \end{aligned} \quad (23)$$

a result consistent with the fact that E_i and B_{ji} are functionals of the X's. We thus see that, in spite of the absence of the usual commutator term $\mathrm{i}[A_\mu, A_\nu]$ of non-Abelian gauge theories, in our case the field strengths transform like non-Abelian ones. We recall that these are all consequences of the matrix coordinates of D0-branes. Finally, by a similar reason, the vanishing of the second term of (10), both sides of (19) transform identically.

An equation of motion similar to (18) is considered in [11,12] as part of the similarities between the dynamics of D0-branes and bound states of quarks and QCD strings [11–13]. The point is that the center-of-mass dynamics of D0-branes is not affected by the non-Abelian sector of the background, i.e., the center-of-mass is "white" with respect to the $SU(N)$ sector of $U(N)$. The center-of-mass coordinates and momenta are defined by

$$X^i_{\text{c.m.}} \equiv \frac{1}{N} \mathrm{Tr} X^i, \quad P^i_{\text{c.m.}} \equiv \mathrm{Tr} P^i_X, \quad (24)$$

where we are using the convention $\mathrm{Tr}\mathbf{1}_N = N$. To specify the net charge of a bound state, its dynamics should be studied in zero magnetic and uniform electric fields[3], i.e., $B_{ji} = 0$ and $E_i(X,t) = E_{0i}$; thus these fields do not involve X matrices and contain just the $U(1)$ part. In other words, under gauge transformations E_{0i} and $B_{ji} = 0$ transform to $\tilde{E}_i(X,t) = V^\dagger(X,t) E_{0i} V(X,t) = E_{0i}$ and $\tilde{B}_{ji} = 0$. Thus the action (8) yields the following equation of motion:

$$(N m_0) \ddot{X}^i_{\text{c.m.}} = N q E^i_{0(1)}, \quad (25)$$

in which the subscript (1) emphasizes the $U(1)$ electric field. So the center-of-mass only interacts with the $U(1)$ part of $U(N)$. From the string theory point of view, this observation is based on the simple fact that the $SU(N)$ structure of D0-branes arises just from the internal degrees of freedom inside the bound state.

5 Map to non-Abelian theory

In [7] a map between field configurations of non-commutative and ordinary gauge theories is introduced, which preserves the gauge equivalence relation. It is emphasized that the map is not an isomorphism between the gauge groups. It will be interesting to study the properties of the map between non-Abelian gauge theory and gauge theory associated with matrix coordinates of D0-branes; on the one side the quantum theory of matrix fields, and on the other side the quantum mechanics of matrix coordinates.

[2] D_t is absent in the definition of E_i, because, the combination $\mathrm{i}[a_0, A_i]$ has been absorbed to produce $D_t X^j$ for both parts of B_{ji}

[3] In a non-Abelian gauge theory an uniform electric field can be defined up to a gauge transformation, which is quite well for identification of white (singlet) states

Table 1. Map between non-commutative and ordinary gauge theories

Non-Abelian gauge theory	$\Leftrightarrow$	Electrodynamics on matrix space
$A^\mu(x) = A^\mu_a(x)T^a$	$=$	$A^\mu(X) + (\Lambda(X) + \delta\Lambda(X))$
$F^{\mu\nu}(x) = F^{\mu\nu}_a(x)T^a$	$=$	$F^{\mu\nu}(X)$
$J^\mu(x) = J^\mu_a(x)T^a$	$=$	$D_\tau X^\mu$
$\Lambda(x) = \Lambda_a(x)T^a$	$=$	$\Lambda(X)$

Since in this case we have matrices on both sides, it may be possible to find an isomorphism between all objects involving in the two theories, i.e., dynamical variables and transformation parameters.

It is useful to consider some points in this direction. We may begin by the action

$$S = \int \mathrm{d}^{d+1}x \left(-\bar\psi(\gamma^\mu\partial_\mu + m)\psi - \mathrm{Tr}\left(\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + J_\mu A^\mu\right)\right), \tag{26}$$
$$A^\mu(x) = A^\mu_a(x)T^a, \quad F^{\mu\nu}(x) = F^{\mu\nu}_a(x)T^a, \quad \mu,\nu = t, i,$$

in which the term $J_\mu A^\mu$ is responsible for the interaction, and can be taken in the standard form $J^a_\mu = \mathrm{i}\bar\psi\gamma_\mu T^a\psi$. Gauge invariance specifies the behavior of the current J_μ under the gauge transformations to be $J(x) \to J'(x) = U^\dagger J(x) U$.

Now we can sketch the form of the map between the two theories in Table 1.

Two points should be emphasized. First, in the above we are sketching the relation or map between a field theory and a world-line theory of a particle in a matrix space; like the one that we assume for the relation between field theories and theories living on the world-sheet of strings. Second, though other gauges (like the light-cone one of [12,11]) maybe have some more advantages, here we have assumed that a covariant theory on matrix space is also available; in the above it is needed to define the covariant derivative D_τ along the world-line (see [6] as an example of such a theory). In Table 1 we mention that, firstly, the objects in both sides are matrices, and so the number of degrees of freedom matches. Secondly, field strengths and currents of the two theories transform identically, i.e., in the adjoint representation.

The fate of the map after quantization is interesting. It remains to be understood which correlation functions of the two theories should be put "equal". We leave this for the future.

As a last point, it will be interesting to mention the conceptual relation between the above map, and the ideas concerned in special relativity. Let us take the following general prescription in our physical theories: *the structure of space-time has to be in correspondence with the fields*: fields $\Leftrightarrow$ coordinates. In this way one understands that the space-time coordinates x^μ as well as gauge potentials A^μ behave like a $(d+1)$-vector (spin 1) under the boost transformations. This is just the same idea as in special relativity: to change the picture of space-time so as to be consistent with the Maxwell equations.

Table 2. Analogy of fields and coordinates as treated in the text

Field	Space-time coordinates	Theory
Photon A^μ	x^μ	Electrodynamics
Fermion ψ	θ, $\bar\theta$	Supersymmetric
Gluon A^μ_a	X^μ_a	Chromodynamics?

Also in this way supersymmetry is a natural continuation of the special relativity program: Adding the spin 1/2 sector to the coordinates of space-time, as the representative of the fermions of nature. This leads one to the super-space formulation of the supersymmetric theories, and in the same way fermions are introduced into the bosonic string theory.

Now, what modifications may be thought of if nature has non-Abelian (non-commutative) gauge fields? In the present view of nature non-Abelian gauge fields cannot form spatially long coherent states; they are confined or too heavy. But the picture may be changed inside those regions of space-time where such fields are non-zero. In fact recent developments in string theory sound this change and it is understood that non-commutative coordinates and non-Abelian gauge fields are two sides of the same coin. We may summarize the above discussion in Table 2 [12,11].

6 Finite-N non-commutative phenomenology

Recently non-commutative field theories have attracted a great deal of interest. Most of these kinds of studies concern theories which are defined on spaces whose coordinates satisfy the algebra $[\hat x^\mu, \hat x^\nu] = \mathrm{i}\theta^{\mu\nu}$. This algebra is satisfied just by $\infty \times \infty$ matrices, and as a consequence, the concerned non-commutativities should be assumed in all regions of the space. Also, generally in these spaces one should expect violation of Lorentz invariance.

In the case we have for D0-branes, the non-commutativity of matrix coordinates is "confined" inside the bound state, and so it appears to be different, and maybe more interesting. How can we probe this non-commutativity? The answer is gained simply through "the response of non-commutativity to the external probes." The dynamics of D0-branes in a background of curved metric $G_{\mu\nu}(x,t)$ and the 1-form (RR) field $A_\mu(x,t)$ can be given to lowest orders by (we are not being very precise about indices and coefficients) [4,5]

$$S = \int \mathrm{d}t \mathrm{Tr}\bigg(\frac{m}{2} G_{ij}(X,t) D_t X^i D_t X^j + qG_{ij}(X,t)A^i(X,t)D_tX^j - qA_t(X,t) + mG(X,t)G(X,t)\frac{[X,X]^2}{l^4} + (1 - G_{00}(X,t)) + \cdots \bigg). \tag{27}$$

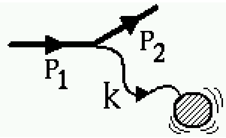

Fig. 1. Long wavelength scattering: sub-structure is not seen

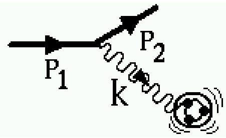

Fig. 2. Short wavelength scattering: non-commutativity is probed

We again mention that the backgrounds $G_{\mu\nu}(x,t)$ and $A_\mu(x,t)$ appear in the action by a functional dependence on the matrix coordinates X. In fact this is the key of the answer to the question "How to probe non-commutativity?" In a Fourier expansion of the background we find

$$A(X,t) = \sum_k \bar{A}(k,t)\mathrm{e}^{\mathrm{i}k_i X^i},$$
$$G(X,t) = \sum_k \bar{G}(k,t)\mathrm{e}^{\mathrm{i}k_i X^i}, \quad (28)$$

in which $\bar{A}(k,t)$ and $\bar{G}(k,t)$ are the Fourier components of the fields $A(x,t)$ and $G(x,t)$ respectively; i.e., fields by ordinary coordinates. One can imagine the scattering processes which are designed to probe inside the bound states. As in every other scattering process we have two regimes:

(1) long wavelength,
(2) short wavelength.

In the small k (long wavelength) regime, the fields A_μ and $G_{\mu\nu}$ do not involve X matrices mainly, and the fields will appear to be nearly constant inside the bound state. So in this regime non-commutativity will not be seen; see Fig. 1.

In the large k (short wavelength) regime, the fields depend on the coordinates X, and so the sub-structure responsible for non-commutativity should be probed; see Fig. 2. As we recalled previously, in fact it is understood that the non-commutativity of D0-brane coordinates is a consequence of the strings which are stretched between D0-branes. So, by these kinds of scattering processes one should be able to probe both D0-branes (as point-like objects), and the strings stretched between them.

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